Exercise - 2.4

Complete Solution by Mubashar Siddique

Question 1: Without using calculator, evaluate the following

(i) $\log_2(18) - \log_2(9)\ \log_2\left(\frac{18}{9}\right) = \log_2(2) = 1$

Answer: 1

(ii) $\log_2(64) + \log_2(2)\ \log_2(64 \times 2) = \log_2(128) = 7$

Answer: 7

(iii) $\frac{1}{3}\log_3(8) - \log_3(18)\ \log_3(2) - \log_3(18) = \log_3\left(\frac{1}{9}\right) = -2$

Answer: -2

(iv) $2\log(2) + \log(25)\ \log(2^2) + \log(25) = \log(100) = 2$

Answer: 2

(v) $\frac{1}{3}\log_4(64) + 2\log_5(25)\ \frac{1}{3}\log_4(4^3) + 2\log_5(5^2) = 1 + 4 = 5$

Answer: 5

(vi) $\log_3(12) + \log_3(0.25)\ \log_3(12 \times 0.25) = \log_3(3) = 1$

Answer: 1

Question 2: Write the following as a single logarithm

(i) $\frac{1}{2}\log(25) + 2\log(3)$

Solution: $\log(5) + \log(9) = \log(45)$

Final Answer: $\log(45)$

(ii) $\log(9) - \log\left(\frac{1}{3}\right)$

Solution: $\log\left(\frac{9}{1/3}\right) = \log(27)$

Final Answer: $\log(27)$

(iii) $\log_5(b^2) \times \log_a(5^3)$
Solution: $2\log_5(b) \times 3\log_a(5) = 6\log_a(b)$
Final Answer: $6\log_a(b)$

(iv) $2\log_3(x) + \log_3(y)$
Solution: $\log_3(x^2) + \log_3(y) = \log_3(x^2y)$
Final Answer: $\log_3(x^2y)$

(v) $4\log_5(x) - \log_5(y) + \log_5(z)$
Solution: $\log_5(x^4) + \log_5(z) - \log_5(y) = \log_5\left(\frac{x^4z}{y}\right)$
Final Answer: $\log_5\left(\frac{x^4z}{y}\right)$

(vi) $2\ln(a) + 3\ln(b) - 4\ln(c)$
Solution: $\ln(a^2) + \ln(b^3) - \ln(c^4) = \ln\left(\frac{a^2b^3}{c^4}\right)$
Final Answer: $\ln\left(\frac{a^2b^3}{c^4}\right)$

Question 3: Expand the following using laws of logarithms

(i) $\log\left(\frac{11}{5}\right)$

$$\log\left(\frac{11}{5}\right) = \log 11 - \log 5$$

(ii) $\log_5(8a^6)$

$$\log_5\left(8a^6\right) = \log_5 8 + \log_5(a^6) = \log_5 8 + 6\log_5 a$$

(iii) $\ln(a^2bc)$

$$\ln\left(a^2bc\right) = \ln(a^2) + \ln b + \ln c = 2\ln a + \ln b + \ln c$$

(iv) $\log_9\left(\frac{z}{xy}\right)$

$$\log_9\left(\frac{z}{xy}\right) = \log_9 z - \log_9(xy) = \log_9 z - (\log_9 x + \log_9 y)$$

(v) $\frac{1}{3}\ln(16x^3)$

$$\tfrac{1}{3}\ln(16x^3) = \tfrac{1}{3}(\ln 16 + \ln(x^3)) = \tfrac{1}{3}\ln 16 + \ln x$$

(vi) $\log_2\left(\frac{1-a}{b}\right)^5$

$$\log_2\left(\frac{1-a}{b}\right)^5 = 5\log_2\left(\frac{1-a}{b}\right) = 5\big(\log_2(1-a) - \log_2 b\big)$$

Question 4: Find the value of x in the following equations

(i) $\log 2 + \log x = 1$

$\log(2x) = 1 \quad \Rightarrow \quad 2x = 10 \quad \Rightarrow \quad x = 5.$

$\boxed{x = 5}$

(ii) $\log_2 x + \log_2 8 = 5$

$\log_2 x + 3 = 5 \quad \Rightarrow \quad \log_2 x = 2 \quad \Rightarrow \quad x = 4.$

$\boxed{x = 4}$

(iii) $(81)^x = (243)^{x+2}$

$3^{4x} = 3^{5x+10} \quad \Rightarrow \quad 4x = 5x + 10 \quad \Rightarrow \quad x = -10.$

$\boxed{x = -10}$

(iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

$3^{-3(x-6)} = 3^3 \quad \Rightarrow \quad -3x + 18 = 3 \quad \Rightarrow \quad x = 5.$

$\boxed{x = 5}$

(v) $\log(5x - 10) = 2$

$5x - 10 = 100 \quad \Rightarrow \quad 5x = 110 \quad \Rightarrow \quad x = 22.$

$\boxed{x = 22}$

(vi) $\log_2(x+1) - \log_2(x-4) = 2$

$\log_2\left(\frac{x+1}{x-4}\right) = 2 \quad \Rightarrow \quad \frac{x+1}{x-4} = 4 \quad \Rightarrow \quad x = \frac{17}{3}.$

$\boxed{x = \frac{17}{3}}$

Q5. Find the values of the following with the help of logarithm table

Formula:

$$\log\left(\frac{A \times B}{C}\right) = \log A + \log B - \log C$$

$$\text{and} \quad \log(A \times B \times C) = \log A + \log B + \log C$$

(i) $\dfrac{3.68 \times 4.21}{5.234}$

$$\log(3.68 \times 4.21) - \log(5.234)$$

$$= (\log 3.68 + \log 4.21) - \log 5.234$$

Using logarithm table:

$$\log 3.68 = 0.5658, \quad \log 4.21 = 0.6242, \quad \log 5.234 = 0.7190$$

$$\therefore \log(\text{result}) = (0.5658 + 0.6242) - 0.7190 = 0.4710$$

$$\text{Result} = \text{Antilog}(0.4710) = 2.96$$

(ii) $4.67 \times 2.11 \times 2.397$

$$\log(\text{result}) = \log 4.67 + \log 2.11 + \log 2.397$$

Using logarithm table:

$$\log 4.67 = 0.6693, \quad \log 2.11 = 0.3243, \quad \log 2.397 = 0.3794$$

$$\log(\text{result}) = 0.6693 + 0.3243 + 0.3794 = 1.3730$$

$$\text{Result} = \text{Antilog}(1.3730) = 23.65$$

(iii) $\dfrac{(20.46)^2 \times (2.4122)}{754.3}$

$$\log(\text{result}) = 2\log(20.46) + \log(2.4122) - \log(754.3)$$

Using logarithm table:

$$\log 20.46 = 1.3116, \quad \log 2.4122 = 0.3822, \quad \log 754.3 = 2.8774$$

$$\log(\text{result}) = 2(1.3116) + 0.3822 - 2.8774 = 0.1280$$

$$\text{Result} = \text{Antilog}(0.1280) = 1.34$$

(iv) $\dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

$$\log(\text{result}) = \tfrac{1}{3}\log(9.364) + \log(21.64) - \log(3.21)$$

Using logarithm table:

$$\log 9.364 = 0.9719, \quad \log 21.64 = 1.3354, \quad \log 3.21 = 0.5065$$

$$\log(\text{result}) = \tfrac{1}{3}(0.9719) + 1.3354 - 0.5065$$
$$= 0.3239 + 1.3354 - 0.5065 = 1.1528$$
$$\text{Result} = \text{Antilog}(1.1528) = 14.2$$

Exercise 2.4 – Questions 6, 7 & 8

Q6. Earthquake Magnitude

Question: The formula to measure the magnitude of earthquakes is given by:

$$M = \log_{10}\left(\frac{A}{A_0}\right)$$

where A is the amplitude and A_0 is the reference amplitude. If $A = 10,000$ and $A_0 = 10$, find the magnitude of the earthquake.

Solution:

$$M = \log_{10}\left(\frac{10000}{10}\right) = \log_{10}(1000)$$

Since $1000 = 10^3$,

$$M = 3$$

$$\boxed{M = 3}$$

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Q7. Investment Growth

Question: Abdullah invested Rs. $100,000$ in a saving scheme. The investment gains interest at the rate of 5% per annum, so that the total value of this investment after t years is given by:

$$y = 100,000(1.05)^t, \quad t \geq 0$$

Find after how many years the investment will be **double**.

Solution: When the investment doubles, $y = 200,000$. Substituting:

$$200,000 = 100,000(1.05)^t$$

Divide through by $100,000$:

$$2 = (1.05)^t$$

Take natural logarithm on both sides:

$$\ln 2 = t \ln(1.05)$$

$$t = \frac{\ln 2}{\ln(1.05)}$$

Substitute values:

$$\ln 2 = 0.693147, \quad \ln(1.05) = 0.048790$$

$$t = \frac{0.693147}{0.048790} = 14.21$$

$$\boxed{t \approx 14.21 \text{ years}}$$

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Q8. Temperature Change with Altitude

Question: Huria is hiking up a mountain where the temperature decreases by 3% (a factor of 0.97) for every 100 metres of altitude gained. The initial temperature at sea level (T_i) is 20°C.

The formula is:

$$T = T_i \times 0.97^{\frac{h}{100}}$$

Calculate the temperature at an altitude of $h = 500$ metres.

Solution: Substitute values into the formula:

$$T = 20 \times 0.97^{\frac{500}{100}} = 20 \times 0.97^5$$

Step-by-step calculation of 0.97^5:

$$0.97^2 = 0.9409, \quad 0.97^3 = 0.912673, \quad 0.97^4 = 0.885293, \quad 0.97^5 = 0.858734$$

$$T = 20 \times 0.858734 = 17.1747$$

$$\boxed{T \approx 17.17°\text{C}}$$