Exercise – 4.3

Solution By Mubashar Siddique

Question 1 Find HCF by factorization method.

$i)\ 21x^2y, \quad 35xy^2$

$21x^2y = 3\times 7\times x\times x\times y$

$35xy^2 = 5\times 7\times x\times y\times y$

$HCF = \text{Product Of Common Factors}$

$HCF = 7\times x\times y$

$HCF = 7xy \quad Ans$

$ii)\ 4x^2-9y^2, \quad 2x^2-3xy$

Since

$(2x)^2-(3y)^2 \qquad \therefore a^2-b^2=(a+b)(a-b)$

$(2x-3y)(2x+3y)$

$4x^2-9y^2=(2x-3y)(2x+3y)$

$2x^2-3xy = x(2x-3y)$

$HCF = \text{Product Of Common Factors}$

$HCF = 2x-3y \quad Ans$

$iii)\ x^3-1, \quad x^2+x+1$

$x^3-1=(x-1)(x^2+x+1)$

$x^2+x+1=x^2+x+1$

$HCF = \text{Product Of Common Factors}$

$HCF = x^2+x+1 \quad Ans$

$iv)\ a^3+2a^2-3a, \quad 2a^3+5a^2-3a$

After extracting their factors

$a^3+2a^2-3a = a(a+3)(a-1)$

$2a^3+5a^2-3a = a(a+3)(2a-1)$

$HCF = \text{Product Of Common Factors}$

$HCF = a(a+3) \quad Ans$

$v)\ t^2-3t-4,\ t^2+5t+4,\ t^2-1$

After extracting thier factors

$t^2-3t-4=(t+1)(t-4)$

$t^2+5t+4=(t+4)(t+1)$

$t^2-1=(t+1)(t-1)$

$HCF = \text{Product Of Common Factors}$

$HCF = t+1 \quad Ans$

$vi)\ x^2+15x+56, \quad x^2+5x-24, \quad x^2+8x$

$x^2+15x+56 = (x+8)(x+7)$

$x^2+5x-24=(x+8)(x-3)$

$x^2+8x = x(x+8)$

$HCF = \text{Product Of Common Factors}$

$HCF = (x+8) \quad Ans$

Question 2 Find HCF of the following expressions by using division method.

i) $27x^3+9x^2-3x-10, \quad 3x-2$

$$
\begin{array}{r}
9x^2+9x+5 \\
3x-2\,\big)\,27x^3+9x^2-3x-10 \\
\pm 27x^3 \mp 18x^2 \qquad\qquad \\
\hline
27x^2-3x-10 \\
\pm 27x^2 \mp 18x \qquad \\
\hline
15x-10 \\
\pm 15x \mp 10 \\
\hline
0
\end{array}
$$

$HCF = 3x-2 \quad Ans$

ii) $x^3-9x^2+23x-15, \quad x^2-4x+3$

$$
\begin{array}{r}
9x^2+9x+5 \\
x^2-4x+3\,\big)\,x^3-9x^2+23x-15 \\
\pm x^3 \mp 4x^2 \pm 3x \qquad \\
\hline
-5x^2+20x-15 \\
\mp 5x^2 \pm 20x \mp 15 \\
\hline
0
\end{array}
$$

$HCF = x^2-4x+3 \quad Ans$

iii) $2x^3+2x^2+2x+2, \quad 6x^3+12x^2+6x+12$

$$
\begin{array}{r}
9x^2+9x+5 \\
2x^3+2x^2+2x+2\,\big)\,6x^3+12x^2+6x+12 \\
\pm 6x^3 \pm 6x^2 \pm 6x \pm 6 \\
\hline
6x^2 \qquad +6 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
\frac{x}{3} \\
6x^2+6\,\big)\,2x^3+2x^2+2x+2 \\
\pm 2x^3 \qquad \pm 2x \qquad \\
\hline
2x^2 \qquad +2 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
3 \\
2x^2+2\,\big)\,6x^2+6 \\
\pm 6x^2 \pm 6 \\
\hline
0
\end{array}
$$

$HCF = 2x^2+2 = 2(x^2+1)$

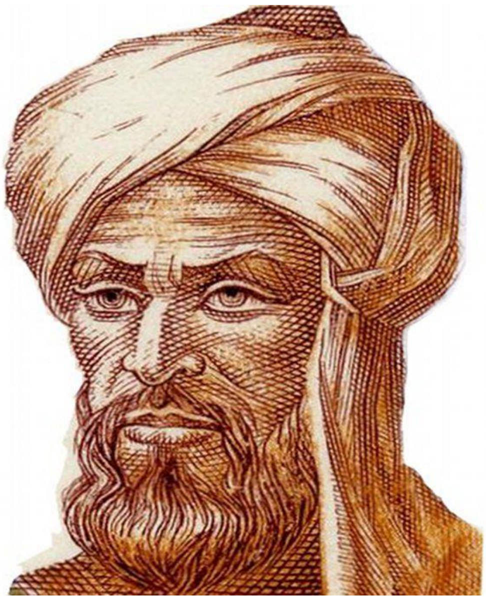
iv) $2x^3 - 4x^2 - 16\,\text{x}, \quad x^3 - 4x, \quad 3x^2 + 6x$

Consider $x^2 + 2x$

$$
\begin{array}{r|l}
 & 9x^2 + 9x + 5 \\
\hline
x^2 + 2x & x^3 - 4x \\
 & \pm x^3 \quad \pm 2x^2 \\
\hline
 & -2x^2 - 4x \\
 & \pm 2x^2 \mp 4x \\
\hline
 & 0 \\
\hline
\end{array}
$$

$$
\begin{array}{r|l}
 & 2x - 8 \\
\hline
x^2 + 2x & 2x^3 - 4x^2 - 16x \\
 & \pm 2x^3 \pm 4x^2 \\
\hline
 & -8x^2 - 16x \\
 & \mp 8x^2 \mp 16x \\
\hline
 & 0 \\
\hline
\end{array}
$$

$HCF = x^2 + 2x = x(x+2)$ *Ans*



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Question 3 Find LCM of the following expressions using prime factorization method.

$i)\ 2a^2b,\quad 4ab^2,\quad 6ab$

$2a^2b = 2\times a\times a\times b$

$4ab^2 = 2\times 2\times a\times b\times b$

$6ab = 2\times 3\times a\times b$

$Common\ Factors\ = 2\times a\times b$

$Non\ Common\ Factors = 2\times 3\times a\times b$

$LCM\ = Common\times Non\ Common$

$LCM\ = 12a^2b^2\quad Ans$

$ii)\ x^2+\mathrm{x},\quad x^3+x^2$

$x^2+\mathrm{x} = x(\mathrm{x}+1)$

$x^3+x^2 = x\times x(x+1)$

$Common\ Factors\ = x(x+1)$

$Non\ Common\ Factors = x$

$LCM\ = Common\times Non\ Common$

$LCM\ = x^2(x+1)\quad Ans$

$iii)\ a^2-4\mathrm{a}+4,\quad a^2-2a$

$a^2-4\mathrm{a}+4 = (\mathrm{a}-2)(\mathrm{a}-2)$

$a^2-2a = a(\mathrm{a}-2)$

$Common\ Factors\ = (\mathrm{a}-2)$

$Non\ Common\ Factors = a(a-2)$

$LCM\ = Common\times Non\ Common$

$LCM\ = a(\mathrm{a}-2)\times(\mathrm{a}-2)\quad Ans$

$iv)\ x^4-16,\quad x^3-4x$

$x^4-16 = (\mathrm{x}^2-4)(\mathrm{x}^2+4) = (\mathrm{x}-2)(\mathrm{x}+2)(\mathrm{x}^2+4)$

$x^3-4x = x(\mathrm{x}^2-4) = \mathrm{x}(\mathrm{x}-2)(\mathrm{x}+2)$

$Common\ Factors\ = (\mathrm{x}-2)(\mathrm{x}+2)$

$Non\ Common\ Factors = x(\mathrm{x}^2+4)$

$LCM\ = Common\times Non\ Common$

$LCM\ = x(\mathrm{x}-2)(\mathrm{x}+2)(\mathrm{x}^2+4)\quad Ans$

$v)\ 16-4x^2,\quad x^2+x-6,\quad 4-x^2$

$16-4x^2 = 4(4-\mathrm{x}^2) = (2-\mathrm{x})(2+\mathrm{x})$

$x^2+x-6 = \mathrm{x}^2+3x-2x-6 = (\mathrm{x}+3)(\mathrm{x}-2) = -(\mathrm{x}+3)(2-\mathrm{x})$

$4-x^2 = (2-x)(2+x)$

$Common\ Factors\ = (2-\mathrm{x})(2+\mathrm{x})$

$Non\ Common\ Factors = -4(\mathrm{x}+3)$

$LCM\ = Common\times Non\ Common$

$LCM\ = (2-\mathrm{x})(2+\mathrm{x})\times -4(\mathrm{x}+3)$

$LCM\ = 4(\mathrm{x}^2-4)(\mathrm{x}+3)\quad Ans$

Q4) *The HCF of two polynomials is* $y-7$ *their LCM is* $y^3-10y^2+11y+70$. *If one of the polynomials is* $y^2-5y-14$, *find the other.*

$HCF = y-7$

$LCM = y^3-10y^2+11y+70$

$p(y) = y^2-5y-14$; $q(y) = ?$

As we know: $p(y) \times q(y) = HCF \times LCM$

Using above formula

$$q(y)=\frac{(y-7)(y^3-10y^2+11y+70)}{y^2-5y-14}$$

$$q(y)=\frac{(y-7)(y^3-10y^2+11y+70)}{(y-7)(y+2)}$$

$$q(y)=\frac{(y^3-10y^2+11y+70)}{(y+2)}$$

$$\begin{array}{r|l|l}
 & y^3-10y^2+11y+70 & \\
y+2 & \pm y^3 \pm 2y^2 & y^2-12y+35 \\
\hline
 & \quad -12y^2+11y+70 & \\
 & \quad \mp 12y^2 \mp 24y & \\
\hline
 & \qquad 35y+70 & \\
 & \qquad \pm 35y \pm 70 & \\
\hline
 & \qquad\quad 0 &
\end{array}$$

Hence $q(y)=y^2-12y+35$

Q5) *The LCM and HCF of two polynomials* $p(x)$ *and* $q(x)$ *are* $36x^3(x+a)(x^3-a^3)$ *and* $x^2(x-a)$ *respectively.*

If $p(x)=4x^2(x^2-a^2)$ *find* $q(x)$.

$HCF = x^2(x-a)$

$LCM = 36x^3(x+a)(x^3-a^3)$

$p(y) = 4x^2(x^2-a^2)$; $q(y) = ?$

As we know: $p(y) \times q(y) = HCF \times LCM$

Using above formula

$4x^2(x^2-a^2) \times q(y) = x^2(x-a)\times 36x^3(x+a)(x^3-a^3)$

$$q(y)=\frac{x^2(x-a)\times 36x^3(x+a)(x^3-a^3)}{4x^2(x^2-a^2)}$$

$q(y)=9x^3(x^3-a^3)$

Hence $q(y)=9x^3(x^3-a^3)$ *Ans*

$Q6)$ *The HCF and LCM of two polynomials is* $(x+a)$ *and* $12x^2(x+a)(x^2-a^2)$ *respectively. Find the product two polynomials.*

$HCF = (x+a)$

$LCM = 12x^2(x+a)(x^2-a^2)$

$p(y)\times q(y) = ?$

As we know: $p(y)\times q(y) = HCF \times LCM$

Using above formula

$p(y)\times q(y) = (x+a)\times 12x^2(x+a)(x^2-a^2)$

$p(y)\times q(y) = 12x^2(x+a)^2(x^2-a^2)$

$p(y)\times q(y) = 12x^2(x+a)^2(x+a)(x-a)$

$p(y)\times q(y) = 12x^2(x+a)^3(x-a)$

Hence $p(y)\times q(y) = 12x^2(x+a)^3(x-a)$

My actions are my only true belongings. I cannot escape the consequences of my actions. My actions are the ground upon which I stand.

Thich Nhat Hanh

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