

Exercise – 4.2

Solution By Mubashar Siddique

Question 1 Factorize each of the following expression

$$\begin{aligned}
 \text{i) } 4x^4 + 81y^4 &= (2x^2)^2 + (9y^2)^2 = (2x^2)^2 + (9y^2)^2 + 2(2x^2)(9y^2) - 2(2x^2)(9y^2) \\
 &= (2x^2 + 9y^2)^2 - 36x^2y^2 = (2x^2 + 9y^2)^2 - (6xy)^2 \\
 &= (2x^2 + 9y^2 - 6xy)(2x^2 + 9y^2 + 6xy)
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } a^4 + 64b^4 &= (a^2)^2 + (8b^2)^2 = (a^2)^2 + (8b^2)^2 + 2(a^2)(8b^2) - 2(a^2)(8b^2) \\
 &= (a^2 + 8b^2)^2 - 16a^2b^2 = (a^2 + 8b^2)^2 - (4ab)^2 \\
 &= (a^2 + 8b^2 + 4ab)(a^2 + 8b^2 - 4ab)
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } x^4 + 4x^2 + 16 &= x^4 + 8x^2 - 4x^2 + 16 = x^4 + 8x^2 + 16 - 4x^2 \\
 &= (x^2 + 4)^2 - (2x)^2 \\
 &= (x^2 + 4 - 2x)(x^2 + 4 + 2x)
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } x^4 - 16x^2 + 1 &= x^4 - 16x^2 + 2x^2 + 1 = x^4 + 2x^2 + 1 - 16x^2 \\
 &= (x^2 + 1)^2 - (4x)^2 \\
 &= (x^2 + 1 - 4x)(x^2 + 1 + 4x)
 \end{aligned}$$

What people say, what people do, and what they say they do are entirely different things.

$$\begin{aligned}
 v) \quad & x^4 - 30x^2y^2 + 9y^4 \\
 &= x^4 - 30x^2y^2 + 6x^2y^2 + 9y^4 = x^4 + 6x^2y^2 + 9y^4 - 36x^2y^2 \\
 &= (x^2 + 3y^2)^2 - (6xy)^2 \\
 &= (x^2 - 6xy + 3y^2)(x^2 + 6xy + 3y^2)
 \end{aligned}$$

$$\begin{aligned}
 vi) \quad & x^4 - 7x^2y^2 + y^4 \\
 &= x^4 - 9x^2y^2 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - 9x^2y^2 \\
 &= (x^2 + y^2)^2 - (3xy)^2 \\
 &= (x^2 - 3xy + y^2)(x^2 + 3xy + y^2)
 \end{aligned}$$

Question 2 Factorize each of the following expression

$$\begin{aligned}
 i) \quad & (x+1)(x+2)(x+3)(x+1)+1 \\
 &= (x+1)(x+4)(x+2)(x+3)+1 \\
 &= (x^2+5x+4)(x^2+5x+6)+1 \\
 &= (y+4)(y+6)+1 = y^2+10y+24+1 = y^2+10y+25 \\
 &= (y+5)^2 = (x^2+5x+5)^2
 \end{aligned}$$

$$\begin{aligned}
 ii) \quad & (2x^2+7x+3)(2x^2+7x+5)+1 \\
 &= (2x^2+7x+3)(2x^2+7x+5)+1 \\
 &= (y+3)(y+5)+1 = y^2+8y+15+1 = y^2+8y+16 \\
 &= (y+4)^2 = (2x^2+7x+4)^2
 \end{aligned}$$

$$\begin{aligned}
 iii) \quad & (x+2)(x-7)(x-4)(x-1)+17 \\
 &= (x+2)(x-7)(x-4)(x-1)+17 \\
 &= (x^2-5x+14)(x^2-5x+4)+17 \\
 &= (y-14)(y+14)+17 = y^2-10y-56+17 = y^2-10y-39 \\
 &= y^2-13y+3y-39 = y(y-13)+3(y-13) \\
 &= (y-13)(y+3) = (x^2-5x-13)(x^2-5x+13)
 \end{aligned}$$

*Someone who points out
your flaws to you is not
necessarily your enemy.
Someone who speaks of
your virtues is not
necessarily your friend.*

$$\begin{aligned}
 \text{iv) } & (3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3 \\
 &= (3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3 \\
 &= (y+3)(y+5) - 3 = y^2 + 8y + 15 - 3 = y^2 + 8y + 12 \\
 &= y^2 + 6y + 2y + 12 = y(x+6) + 2(y+6) \\
 &= (y+6)(y+2) = (3x^2 + 5x + 6)(3x^2 + 5x + 2)
 \end{aligned}$$

$$\begin{aligned}
 \text{v) } & (x+1)(x+6)(x+2)(x+3) - 3x^2 \\
 &= (x+1)(x+6)(x+2)(x+3) - 3x^2 \\
 &= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2 \\
 &= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2 \\
 &= (y+7x)(y+5x) - 3x^2 = y^2 + 12xy + 35x^2 - 3x^2 \\
 &= y^2 + 12xy + 32x^2 = y^2 + 8xy + 4xy + 32x^2 = y(y+8x) + 4x(y+8x) \\
 &= (y+8x)(y+4x) = (x^2 + 8x + 6)(x^2 + 4x + 6)
 \end{aligned}$$

$$\begin{aligned}
 \text{vi) } & (x+1)(x+2)(x-1)(x-2) - 16x^2 \\
 &= (x+1)(x+2)(x-1)(x-2) - 16x^2 \\
 &= (x^2 + 3x + 2)(x^2 - 5x + 2) - 16x^2 \\
 &= (x^2 + 2 + 3x)(x^2 + 2 - 3x) - 16x^2 \\
 &= (y+3x)(y-3x) - 16x^2 = y^2 - 9y^2 - 16x^2 = -25x^2 \\
 &= (y-5x)(y+5x) = (x^2 - 5x + 2)(x^2 + 5x + 2)
 \end{aligned}$$

Remember!

$$\begin{aligned}
 (a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 (a-b)^3 &= a^3 - 3a^2b + 3ab^2 - b^3
 \end{aligned}$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Question 3 Factorize

$ \begin{aligned} \text{(i) } & 8x^3 + 12^2 + 6x + 1 \\ &= (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3 \\ &= (2x+1)^3 \end{aligned} $	$ \begin{aligned} \text{(iii) } & x^3 + 18x^2y + 108xy^2 + 216y^3 \\ &= (x)^3 + 3(x)^2(6y) + 3(x)(6)^2 + (6y)^3 \\ &= (x+6y)^3 \end{aligned} $
$ \begin{aligned} \text{(ii) } & 27a^3 + 108a^2b + 144ab^2 + 64b^3 \\ &= (3a)^3 + 3(3a)^2(4ab) + 3(3a)(4ab)^2 + (4b)^3 \\ &= (3a+4b)^3 \end{aligned} $	$ \begin{aligned} \text{(iv) } & 8x^3 - 125y^3 + 150xy^2 - 60x^2y \\ &= (2x)^3 + (-5y)^3 + 3(2x)^2(-5y) + 3(2x)(-5y)^2 + (-5y)^3 \\ &= (2x-5y)^3 \end{aligned} $

Question 4 Factorize

$\begin{aligned} & (i) 125a^3 - 1 \\ & = (5a)^3 - (1)^3 \\ & = (5a - 1) \left[(5a)^2 + (5a)(1) + (1)^2 \right] \\ & = (5a - 1)(25a^2 + 5a + 1) \end{aligned}$	$\begin{aligned} & (iv) 343x^3 + 216 \\ & = (7x)^3 + (6)^3 \\ & = (7x + 6) \left[(7x)^2 - (7x)(6) + (6)^2 \right] \\ & = (7x + 6)(49x^2 - 42x + 36) \end{aligned}$
$\begin{aligned} & (ii) 64x^3 + 125 \\ & = (4x)^3 + (5)^3 \\ & = (4x + 5) \left[(4x)^2 - (4x)(5) + (5)^2 \right] \\ & = (4x + 5)(16x^2 - 20x + 25) \end{aligned}$	$\begin{aligned} & (v) 27 - 512y^3 \\ & = (3)^3 - (8y)^3 \\ & = (3 - 8y) \left[(3)^2 + (3)(8y) + (8y)^2 \right] \\ & = (3 - 8y)(9 + 24y + 64y^2) \end{aligned}$
$\begin{aligned} & (iii) 1000a^3 + 1 \\ & = (10a)^3 + (1)^3 \\ & = (10a + 1) \left[(10a)^2 - (10a)(1) + (1)^2 \right] \\ & = (10a + 1)(100a^2 - 10a + 1) \end{aligned}$	$\begin{aligned} & (vi) x^6 - 27 \\ & = (x^2)^3 - (3)^3 \\ & = (x^2 - 3) \left[(x^2)^2 + (x^2)(3) + (3)^2 \right] \\ & = (x^2 - 3)(x^4 + 3x^2 + 9) \end{aligned}$

These notes are prepared by **Mubashar Siddique (BS Mathematics)** and published on this website.

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