Exercise - 1.2

Complete Solution by Mubashar Siddique

Question 1: Rationalize the denominator

Q1 (i): $\dfrac{13}{4+\sqrt{3}}$

$$\frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{13(4-\sqrt{3})}{16-3} = 4-\sqrt{3}$$

Answer: $4-\sqrt{3}$

Q1 (ii): $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}+\sqrt{15}}{3}$$

Answer: $\dfrac{\sqrt{6}+\sqrt{15}}{3}$

Q1 (iii): $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

$$\frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}-\sqrt{5}}{5}$$

Answer: $\dfrac{\sqrt{10}-\sqrt{5}}{5}$

Q1 (iv): $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} = \frac{(6-4\sqrt{2})^2}{36-(4\sqrt{2})^2} = \frac{36-48\sqrt{2}+32}{4} = 17-12\sqrt{2}$$

Answer: $17-12\sqrt{2}$

Q1 (v): $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} = 5-2\sqrt{6}$$

Answer: $5-2\sqrt{6}$

Q1 (vi): $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2} = 2\sqrt{3}(\sqrt{7}-\sqrt{5})$$

Answer: $2\sqrt{3}(\sqrt{7}-\sqrt{5})$

Question 2: Simplify the following

Q2 (i): $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{3^4}{2^4}\right)^{-\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} = \frac{2^3}{3^3} = \frac{8}{27}$$

Answer: $\dfrac{8}{27}$

Q2 (ii): Expression

$$\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27} = \frac{2^4}{3^2} \times \frac{3^6}{2^6} \times \frac{2^4}{3^3} = 12$$

Answer: 12

Q2 (iii): $(0.027)^{-\frac{1}{3}}$

$$(0.027)^{-\frac{1}{3}} = \left(\frac{27}{1000}\right)^{-\frac{1}{3}} = \left(\frac{1000}{27}\right)^{\frac{1}{3}} = \frac{10}{3}$$

Answer: $\dfrac{10}{3}$

Q2 (iv): Root expression

$$\sqrt[7]{\frac{x^{14}y^{21}z^{35}}{y^{14}z^{7}}} = (x^{14}y^{7}z^{28})^{\frac{1}{7}} = x^2yz^4$$

Answer: x^2yz^4

Q2 (v)

$$\frac{5\cdot 25^{n+1} - 25\cdot 5^{2n}}{5\cdot 5^{2n+3} - 25^{n+1}} = \frac{5^{2n+2}(4)}{5^{2n+2}(24)} = \frac{1}{6}$$

Answer: $\dfrac{1}{6}$

Q2 (vi)

$$\frac{16+20(4^{2x})}{2^{x-3}\cdot 8^{x+2}} = \frac{2^{4x}(36)}{2^{4x+3}} = \frac{36}{8} = \frac{9}{2}$$

Answer: $\frac{9}{2}$

Q2 (vii)

$$(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}} = \frac{1}{2^4} \div \frac{1}{3^3} = \frac{27}{16}$$

Answer: $\frac{27}{16}$

Q2 (viii)

$$\frac{3^n\cdot 9^{n+1}}{3^{n-1}\cdot 9^{n-1}} = \frac{3^{3n+2}}{3^{3n-3}} = 3^5 = 243$$

Answer: 243

Q2 (ix)

$$\frac{5^{n+3}-6\cdot 5^{n+1}}{9\cdot 5^n - 4\cdot 5^n} = \frac{5^n(125-30)}{5^n(5)} = 19$$

Answer: 19

Question 3

If $x = 3+\sqrt{8}$, find the following:

(i) $x+\frac{1}{x}$

Solution:

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}}\cdot\frac{3-\sqrt{8}}{3-\sqrt{8}} = 3-\sqrt{8}$$

$$x+\frac{1}{x} = (3+\sqrt{8})+(3-\sqrt{8}) = 6$$

Answer: 6

(ii) $x - \dfrac{1}{x}$

Solution:

$$x - \frac{1}{x} = (3+\sqrt{8}) - (3-\sqrt{8}) = 2\sqrt{8}$$

Answer: $2\sqrt{8}$

(iii) $x^2 + \dfrac{1}{x^2}$

Solution:

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$x^2 + \frac{1}{x^2} = 6^2 - 2 = 34$$

Answer: 34

(iv) $x^2 - \dfrac{1}{x^2}$

Solution:

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 6 \cdot 2\sqrt{8} = 12\sqrt{8}$$

Answer: $12\sqrt{8}$

Q4

Find rational numbers p and q such that

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}.$$

Rationalize the denominator:

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = \frac{8-3\sqrt{2}}{4+3\sqrt{2}} \cdot \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{(8-3\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}.$$

Expand numerator and denominator:

$$\frac{32 - 24\sqrt{2} - 12\sqrt{2} + 18}{16-18} = \frac{50 - 36\sqrt{2}}{-2} = -25 + 18\sqrt{2}.$$

$$\therefore p = -25, \quad q = 18.$$

Question 5: Simplify the following

(i) $\dfrac{(25)^{\frac{3}{2}} \cdot (243)^{\frac{3}{5}}}{16^{\frac{5}{4}} \cdot 8^{\frac{4}{3}}}$

Solution:

$$\frac{(25)^{\frac{3}{2}} \cdot (243)^{\frac{3}{5}}}{16^{\frac{5}{4}} \cdot 8^{\frac{4}{3}}} = \frac{(5^2)^{\frac{3}{2}} \cdot (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \cdot (2^3)^{\frac{4}{3}}} = \frac{5^3 \cdot 3^3}{2^5 \cdot 2^4} = \frac{125 \cdot 27}{32 \cdot 16} = \frac{3375}{512}$$

Answer: $\dfrac{3375}{512}$

(ii) $\dfrac{54 \cdot \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216 \cdot (3^{2x-1})}$

Solution:

$$\frac{54 \cdot \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216 \cdot (3^{2x-1})} = \frac{27 \cdot 2 \cdot 27^{2x \cdot \frac{1}{3}}}{(3^2)^{x+1} + 6^3 \cdot 3^{2x-1}} = \frac{2 \cdot 3^3 \cdot 3^{2x}}{3^{2x+2} + 2^3 \cdot 3^{2x+2}}$$

$$= \frac{2 \cdot 3^2 \cdot 3^1 \cdot 3^{2x}}{3^{2x+2} + 2^3 \cdot 3^{2x+2}} = \frac{2 \cdot 3 \cdot 3^{2x+2}}{3^{2x+2}(1 + 2^3)} = \frac{6}{9} = \frac{2}{3}$$

Answer: $\dfrac{2}{3}$

(iii) $\sqrt{\dfrac{(216)^{\frac{2}{3}} \cdot (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

Solution:

$$\sqrt{\frac{(216)^{\frac{2}{3}} \cdot (25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}} = \sqrt{\frac{(6^3)^{\frac{2}{3}} \cdot (5^2)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}} = \sqrt{\frac{6^2 \cdot 5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{6^2 \cdot 5}{25^{\frac{3}{2}}}} = \sqrt{\frac{6^2 \cdot 5}{5^3}} = \left(\frac{6^2}{5^2}\right)^{\frac{1}{2}} = \frac{6}{5}$$

Answer: $\dfrac{6}{5}$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right)\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

Solution:

$$\begin{aligned}\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right)\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right) &= a - a^{\frac{2}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}} \\ &\quad + a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{4}{3}} + b^2 \\ &= a + b^2\end{aligned}$$

Answer: $a + b^2$