Unit 1: Real Numbers

Exercise 1.1 — Complete Solution

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Q1. Identify as Rational or Irrational

(i) $2.353535 \in \mathbb{Q}$ (terminating decimal)

(ii) $0.\overline{6} = \frac{2}{3} \in \mathbb{Q}$

(iii) $2.236067\ldots \notin \mathbb{Q}$ (irrational)

(iv) $\sqrt{7} \notin \mathbb{Q}$

(v) $e \notin \mathbb{Q}$

(vi) $\pi \notin \mathbb{Q}$

(vii) $5 + \sqrt{11} \notin \mathbb{Q}$

(viii) $\sqrt{3} + \sqrt{13} \notin \mathbb{Q}$

(ix) $\frac{15}{4} \in \mathbb{Q}$

(x) $(2 - \sqrt{2})(2 + \sqrt{2}) = 2 \in \mathbb{Q}$

Q2. Represent on the Number Line

(i) $\sqrt{2} \approx 1.414$: mark between 1 and 2.

(ii) $\sqrt{3} \approx 1.732$: near 1.73.

(iii) $4\frac{1}{3} = \frac{13}{3} \approx 4.333.$

(iv) $-2\frac{1}{7} = -\frac{15}{7} \approx -2.142.$

(v) $\frac{5}{8} = 0.625.$

(vi) $2\frac{3}{4} = \frac{11}{4} = 2.75.$

Q3. Convert recurring decimals to $\frac{p}{q}$

(i) Let $x = 0.\overline{4}$. Then $10x = 4.\overline{4}$. Subtract: $9x = 4 \implies \boxed{\frac{4}{9}}$.

(ii) Let $x = 0.\overline{37}$. Then $100x = 37.\overline{37}$. Subtract: $99x = 37 \implies \boxed{\frac{37}{99}}$.

(iii) Let $x = 0.\overline{21}$. Then $100x = 21.\overline{21}$. Subtract: $99x = 21 \implies \boxed{\frac{7}{33}}$.

Q4. Properties Used

(i) $(a + 4) + b = a + (4 + b)$ Associative (Addition)

(ii) $\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$ Commutative (Addition)

(iii) $x - x = 0$ Additive Inverse

(iv) $a(b + c) = ab + ac$ Distributive

(v) $16 + 0 = 16$ Additive Identity

(vi) $100 \times 1 = 100$ Multiplicative Identity

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$ Associative (Multiplication)

(viii) $ab = ba$ Commutative (Multiplication)

Q5. Properties of Inequalities

(i) $-3 < -1 \implies 0 < 2$ (Add same number)

(ii) $a < b \implies \frac{1}{a} > \frac{1}{b}$ (Reciprocal)

(iii) $a < b \implies a + c < b + c$ (Addition)

(iv) $ac < bc,\ c > 0 \implies a < b$ (Division by +ve)

(v) $ac < bc,\ c < 0 \implies a > b$ (Division by –ve)

(vi) Either $a > b$, $a = b$, or $a < b$ (Trichotomy)

Q6. Rational Numbers Between Pairs

(i) $\frac{1}{4},\ \frac{1}{3}$: $\frac{13}{48},\ \frac{5}{16}$

(ii) $3,\ 4$: $\frac{13}{4},\ \frac{7}{2}$ $(3.25,\ 3.5)$

(iii) $\frac{3}{5},\ \frac{4}{5}$: $\frac{7}{10},\ \frac{3}{4}$ $(0.7,\ 0.75)$