Exercise – 4.4

By Mubashar Siddique

Question 1 Find the square root of the following polynomials by factorization method

(*i*) $x^2-8x+16$

$x^2-8x+16=(x)^2-2(x)(4)+(4)^2=(x-4)^2$

$\sqrt{x^2-8x+16}=\sqrt{(x-4)^2}$

$\sqrt{x^2-8x+16}=\pm(x-4)$

(*ii*) $9x^2+12x+4$

$9x^2+12x+4=(3x)^2+2(3x)(2)+(2)^2=(3x+2)^2$

$\sqrt{9x^2+12x+4}=\sqrt{(3x+2)^2}$

$\sqrt{9x^2+12x+4}=\pm(3x+2)$

(*iii*) $36a^2+84a+49$

$36a^2+84a+49=(6a)^2+2(6a)(7)^2=(6a+7)^2$

$\sqrt{36a^2+84a+49}=\sqrt{(6a+7)^2}$

$\sqrt{36a^2+84a+49}=\pm(6a+7)$

(*iv*) $64y^2-32y+4$

$64y^2-32y+4=(8y)^2-2(8y)(2)+(2)^2=(8y-2)^2$

$\sqrt{64y^2-32y+4}=\sqrt{(8y-2)^2}$

$\sqrt{64y^2-32y+4}=\pm(8y-2)$

(*v*) $200t^2-120t+18$

$200t^2-120t+18=2[100t^2-60t+9]$

$200t^2-120t+18=2[(10t)^2-2(10t)(3)+(3)^2]=2(10\,t-3)^2$

$\sqrt{200t^2-120t+18}=\sqrt{2(10t-3)^2}$

$\sqrt{200t^2-120t+18}=\pm\sqrt{2}(10t-3)$

(vi) $40x^2+120x+90$

$40x^2+120x+90=10(4x^2+12x+9)=10[(2x)^2+2(2x)(3)+(3)^2]$

$40x^2+120x+90=10(2x+3)^2$

$\sqrt{40x^2+120x+90}=\sqrt{10(2x+3)^2}$

$\sqrt{40x^2+120x+90}=\pm\sqrt{10}(2x+3)$

Question 2 Find the square root of the following polynomials by division method

(*i*) $\sqrt{4x^4-28x^3+37x^2+42x+9}$

	$2x^2-7x-3$
	$4x^4-28x^3+37x^2+42x+9$
$2x^2$	$\pm 4x^4$
	$-28x^3+37x^2$
$4x^2-7x$	$\mp 28x^3 \pm 49x^2$
	$-12x^2+42x+9$
$4x^2-14x-3$	$\mp 12x^2 \pm 42x \pm 9$
	0

$$\sqrt{4x^4-28x^3+37x^2+42x+9} = \pm(2x^2-7x-3)$$

(*ii*) $\sqrt{121x^4-198x^3-183x^2+216x+144}$

	$11x^2-9x-12$
	$121x^4-198x^3-183x^2+216x+144$
$11x^2$	$\pm 121x^4$
	$-198x^3-183x^2$
$22x^2-9x$	$\mp 198x^3 \pm 81x^2$
	$-264x^2+216x+144$
$22x^2-18x-12$	$\mp 264x^2 \pm 216x \pm 144$
	0

$$\sqrt{121x^4-198x^3-183x^2+216x+144} = \pm(11x^2-9x-12$$

(iii) $\sqrt{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4}$

	$x^2 - 5xy + y^2$
	$x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$
x^2	$\pm x^4$
	$-10x^3y + 27x^2y^2$
$2x^2 - 5xy$	$\mp 10x^3y \pm 25x^2y^2$
	$2x^2y^2 - 10xy^3 + y^4$
$2x^2 - 10xy + y^2$	$\pm 2x^2y^2 \mp 10xy^3 \pm y^4$
	0

$$\sqrt{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4} = \pm(x^2 - 5xy + y^2)$$

(iv) $\sqrt{4x^4 - 12x^3 + 37x^2 - 42x + 49}$

	$2x^2 - 3x + 7$
	$4x^4 - 12x^3 + 37x^2 - 42x + 49$
$2x^2$	$\pm 4x^4$
	$-12x^3 + 37x^2$
$4x^2 - 3x$	$\mp 12x^3 \pm 9x^2$
	$28x^2 - 42x + 49$
$4x^2 - 6x + 7$	$\pm 28x^2 \mp 42x \pm 49$
	0

$$\sqrt{4x^4 - 12x^3 + 37x^2 - 42x + 49} = \pm(2x^2 - 3x + 7)$$

Question 3 An investor's return R(x) in rupees after investing x thousand rupees is given by quadratic expression

$\mathrm{R}(\mathrm{x}) = -x^2 + 6x - 8$ Factorize the expression and find the investment levels that result in zero return.

Solution

$$\mathrm{R}(\mathrm{x}) = -x^2 + 6x - 8 = -x^2 + 4x + 2x - 8 = -x(x-4) + 2(x-4)$$

$$R(x) = (-x+2)(x-4)$$

$$For\ zero\ return\ R(x) = 0\ we\ have\ (-x+2)(x-4) = 0$$

$$-x+2 = 0 \quad or \quad x-4 = 0$$

$$x = 2 \quad or \quad x = 4$$

$$Investment\ levels\ that\ result\ in\ zero\ return\ will\ be\ x = 2\ and\ x = 4$$

Question 4 A company's profit P(x) in rupees from selling x units of a product is modeled by the cubic expression $\mathrm{P}(\mathrm{x}) = x^3 - 15x^2 + 75x - 125$ Find the break-even point (s), where the profit is zero.

Solution

$$\mathrm{P}(\mathrm{x}) = x^3 - 15x^2 + 75x - 125$$

$$P(x) = (x)^3 - 3(x)^2(5) + 3(x)(5)^2 - (5)^3 = (x-5)^3$$

Since profit is zero, using B(x) = 0 we have $(x-5)^3 = 0$

After taking cube root on both sides, we have x = 5

Question 5 The potential energy V(x) in an electric field varies as a cubic function of distance x. given by:

$V(x) = 2\mathrm{x}^3 - 6x^2 + 4x$ Determine where the potential energy is zero.

Solution

$$V(x) = 2\mathrm{x}^3 - 6x^2 + 4x$$

$$V(\mathrm{x}) = 2x(x^2 - 3x + 2) = 2x(\mathrm{x}-2)(\mathrm{x}-1)$$

$$For\ zero\ potential\ energy, u \sin g\ V(\mathrm{x}) = 0\ \mathrm{we\ have}\ 2\,\mathrm{x}(\mathrm{x}-1)(\mathrm{x}-2) = 0$$

$$Then\ x = 0, x = 1, x = 2$$

Question 6 In structural engineering, the deflection Y(x) of a beam is given by $y(x) = 2x^2 - 8x^2 + 6$ this equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection.

Solution

$$y(x) = 2x^2 - 8x^2 + 6$$
$$y(x) = 2(x^2 - 4x + 3)$$
$$y(x) = 2(x^2 - 3x - x + 3)$$
$$y(x) = 2(x-1)(x-3)$$
$$For\ zero\ potential\ deflection, u \sin g\ y(x) = 0\ we\ have\ 2(x-1)(x-3) = 0$$
$$2 \neq 0\ then\ x - 1 = 0\ \ or\ \ x - 3 = 0$$
$$Then\ x = 1, x = 3$$